4. 'Error is as valuable as accuracy in the production of knowledge.' To what extent is this the case in two areas of knowledge?

From kindergarten to retirement, in education, in employment and in life, we as a society often place great value on avoiding error, and being accurate, in particular when making claims of knowledge. Doing so, however, it is easy to overlook the complex relationship between error, accuracy and knowledge. By investigating the questions 'To what extent do error and accuracy exist absolutely?' and 'What roles do error and accuracy play in the production of knowledge?' in mathematics and the natural sciences, it is seen that the relationship between the three is rarely straightforward.

Notions of error and accuracy are commonly referenced in relation to truthfulness. If a statement or prediction aligns with the truth, we may say that it is accurate, whereas if it is contrary to the truth, we may say that it is in error. Thus in this usage, the error or accuracy of a conclusion is considered only in terms of whether that conclusion is true or false, without regard to how that conclusion was arrived at.

To begin with, it is worth considering the extent to which 'error' exists absolutely at all. Unlike more personal or subjective areas of knowledge, both mathematics and the natural sciences have rigorous requirements for objective justification. The natural sciences, focusing on the natural world, require from its theories conformance with empirical real-world observations, justifying conclusions using sense perception, and so a knowledge claim in the natural sciences can easily be erroneous if it does not conform to the real world. Similarly, knowledge claims in mathematics must be rigorously justified, using reason and deductive logic, so it may initially seem that mathematics, too, must conform to a singular objective reality. The criteria for my IB Mathematics internal assessment investigation, for example, call for mathematics that is 'error-free … at all times' [1], and I, like most students, am acutely aware of how easily marks in mathematics can be lost from small missteps, clearly showing the potential for error in logical reasoning in mathematics.

However, such logical reasoning inevitably stems from a set of assumed axioms, whether implicit or explicit. Indeed, it could be argued that any choice of axioms, no matter how seemingly nonsensical, is valid from a pure mathematical standpoint, calling into question the extent to which error can be said to exist absolutely. As an example, a careful change to one axiom of standard Euclidean geometry results in hyperbolic geometry, which is empirically contradicted by nearly every primary school geometry activity, yet is nevertheless fully self-consistent and mathematically correct. In practice, this implies that one must take care when combining knowledge and methodologies from multiple areas, for example when using mathematics in the natural sciences, to ensure conclusions drawn are accurate in their desired contexts, suggesting that error is relative, in that accurate knowledge claims valuable in one area may be considered erroneous and hinder the production of knowledge in another.

Similar difficulties also exist when assessing the accuracy of claims. On the one hand, theorems in mathematics are typically provable using reason, and hence might be considered absolutely accurate. Scientific theories on the other hand, however, are constantly being improved upon, evidenced through developments in physics in the early 20th century. Often concerning itself with precise mathematical relationships, physics might be viewed as the most firm of the natural sciences, and its conclusions the most confident. This confidence was such that in 1894, physicist Albert Michelson opined that 'most of the grand underlying principles [of physics] have been firmly established', quoting another's opinion that 'future truths … are to be looked for in the sixth place of decimals' [2]. In 1900, the physicist to whom this latter opinion is commonly attributed, Lord Kelvin, remarked that only 'two clouds' remain to be addressed in physics [3]. The resolutions to these 'clouds', however – relativity and quantum mechanics – would each engender a paradigm shift, revealing that classical physics, when it appeared to be at its most supported and certain, was in fact not entirely accurate, casting doubt on the certainty of supposedly-accurate scientific knowledge.

Nevertheless, however, as a consequence of the scientific method, scientific theories are by necessity supported by evidence. While classical Newtonian mechanics, for example, may not be entirely accurate, it is nevertheless supported by a wealth of evidence. Indeed, its two aforementioned successors, relativity and quantum mechanics, show that Newtonian mechanics very closely approximates the more accurate theories in ordinary situations. It seems, then, that error and accuracy is not a binary scale, but that it may be more worthwhile to speak of degrees of accuracy on a continuum, with the goal of the natural sciences being to produce theories that are increasingly close to, but never achieving with certainty, absolute accuracy.

If it is difficult for all error to be removed from scientific knowledge claims, then this also raises questions of the extent to which error can contribute to knowledge. One view of knowledge defines it as ‘justified true belief’, and from this definition it follows that any erroneous conclusion, failing to meet the criterion of truth, cannot unreservedly be called knowledge. Identifying the error, however, be it a hypothesis or a flawed theory like Newtonian mechanics, can contribute to knowledge, and indeed is a main component of scientific inquiry. Similarly, although the scientific method is not used, the identification of errors can also contribute to knowledge in mathematics. Consider, for example, the famous four colour theorem which posits that any map can be coloured using only four colours such that no two like-coloured regions touch. In 1879, English mathematician Alfred Kempe published a supposed proof of the conjecture [4], but after standing for eleven years, the proof was revealed to be flawed by fellow English mathematician Percy Heawood [5]. In the same paper, however, Heawood extended Kempe's work to prove the five-colour theorem, demonstrating that errors can contribute to knowledge, not only insofar as the detection of the error is concerned, but also as the foundations for further developments.

Mirroring the role of erroneous beliefs in developing accuracy in mathematics and the natural sciences, partly-erroneous beliefs can also play a role in education in these areas. Beginning year 11, not a few friends remarked that a great deal of senior secondary education seems to consist of explaining that earlier teachings were not entirely faithful to the truth, causing dismay as painstakingly-learned ‘facts’ must be unlearned. Yet it could be argued that such pedagogical ‘white lies’ are in fact necessary for any understanding to develop at all. While reading Terry Pratchett's *The Science of Discworld*, I saw this described as the ‘lie-to-children’, ‘a statement that is false, but which nevertheless leads … towards a more accurate explanation’, which one ‘will only be able to appreciate if … primed with the lie’ [6]. In 1921, Austrian philosopher Ludwig Wittgenstein used the similar concept of a ladder to describe his own propositions, writing that ‘anyone who understands [them] eventually recognizes them as nonsensical, when he has used them – as steps – to climb beyond them’ [7]. For example, a poster in my middle school Science classroom proclaimed that ‘matter cannot be created or destroyed’, a claim later refuted in IB Chemistry, wherein it was revealed that matter is converted into energy in nuclear reactions. However, at a basic level, where notions like the conservation of matter, the ‘solar system’ model of an atom, or the nonexistence of square roots of negative numbers, are used as pedagogical aids, to detail all the intricacies would lack context and defeat the purpose of introducing the aids in the first place: to ease understanding. Hence, in education, one might introduce a simplified model first to develop a basic understanding, then use this knowledge to revise the (now incomplete) original model. Thus it appears that error can be valuable not only in the production of new knowledge, but also in the creation of personal knowledge from existing shared knowledge through education.

Having considered the extent to which error can contribute to knowledge, there may also be situations where accuracy might not contribute to knowledge. As mentioned earlier, in the natural sciences, theories are usually fundamentally justified based on their ability to conform to the truth, however in mathematics, theorems are typically fundamentally justified based on reason. Thus a mathematical proposition may be true, yet its logical justification may easily be invalid. In my mathematics class, for example, it is a recurring joke to write statements like $\frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$. Although the conclusion is true, the justification is absurd, so such a statement could not be described as knowledge – *justified* true belief – and conversely may impair future understanding. For scholars of mathematics like myself, this implies that, even if the conclusion of a mathematical proof is accurate, caution must nevertheless be taken to ensure that every step in that proof is logically valid, or else the proof will be undermined, a principle applicable to other areas of knowledge, indicating that while accuracy can be valuable, it is not itself the only requirement in the production of knowledge.

Through consideration of the relationship between error, accuracy and knowledge, these explorations suggest that the boundaries between error and accuracy are often not well-defined, with some areas having differing concepts of error, and with others the notion of absolute accuracy being difficult to achieve. Furthermore, where a distinction can be drawn, errors can nevertheless contribute to knowledge in some situations, both through their rectification and their use in education, while in other situations accuracy alone may not be sufficient to produce knowledge. Thus, while at least some degree of accuracy may be a requisite for knowledge, error can also be valuable in its production, and its role should not be dismissed.

[Word Count: 1597]

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